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STABILITY OF A SUPERSONIC BOUNDARY LAYER BEHIND A FAN OF RAREFACTION WAVES

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A problem which has recently become increasingly important is determination of the flow characteristics in regions of the interaction of a supersonic boundary layer with flow irregularities such as a fan of rarefaction waves, a shock wave, etc.

For turbulent boundary layers, the most important characteristics are those which give information both on the average flow and on turbulent heat- or mass-transfer parameters. The problem of the effect of rarefaction waves on a turbulent boundary layer was examined in [1], as an example. For laminar boundary layers, the problem of the stability of flows with irregularities is of the greatest interest. In regions of nonuniform flow, the flow being examined is either accelerated or slowed by a pressure gradient. There has not been sufficient study of the effect of pressure gradients on stability characteristics.

Of the theoretical studies, we should mention [2-4]. The authors of these studies used similarity solutions of boundary-layer equations to establish the stabilizing role of a negative pressure gradient at supersonic velocities. It was shown that the effect of the gradient is diminished by intensive cooling of the surface. A negative pressure gradient has a greater stabilizing effect for the second mode of perturbation than for the first mode.

It was noted in [5] that flow near curved surfaces is not self-similar. Also, the authors of [5] used exact boundary-layer equations to study the stability of a supersonic laminar boundary layer during rotation of the flow. They investigated the case of flow over a convex surface, ensuring rotation of the flow at a certain angle with a specified radius of rotation. Stability in this case was evaluated by means of the gradient Reynolds number and was calculated on the basis of the Dan—Lin equations. It was established that, in the case of flow about a convex wall, a supersonic laminar boundary layer is more stable than in the case of an initial boundary layer on a flat wall.

We do not know of any experimental studies of the stability of gradient flows. At the same time, reliable methods have been developed in recent years for studying the stability of nongradient boundary layers with the use of artificially introduced three-dimensional wave packets [6].

The goal of the present study is to experimentally investigate the stability of a supersonic laminar boundary layer behind a fan of rarefaction waves by means of artificial perturbations.

1. The experiments were conducted in supersonic wind tunnel T-325 at the Institute of Theoretical and Applied Mechanics, Siberian Branch of the Soviet Academy of Sciences. The dimensions of the working part were 200 × 200 mm. The Mach number of the incoming flow M = 2.0, while the Reynolds number $Re_1 = 6.5 \cdot 10^6$ m^{-1}. As the model, we used a steel cone with a cylindrical tail section. The angle at the vertex was 10°, and the diameter of the cylindrical part was 38 mm. Perturbations in the flow were recorded with a TPT-4 dc hot-wire anemometer. We used sensors with a tungsten filament 6 μm in diameter and about 1.2 mm long. The measurements were made along the longitudinal coordinate on a line passing through the maximum of the fluctuation distribution with respect to the normal coordinate directly behind the inflection point on the surface.

The model was secured to the rod of a traversing gear at the center of the work part. The model was set at a zero angle of attack and was moved along the flow (along the x coordi-

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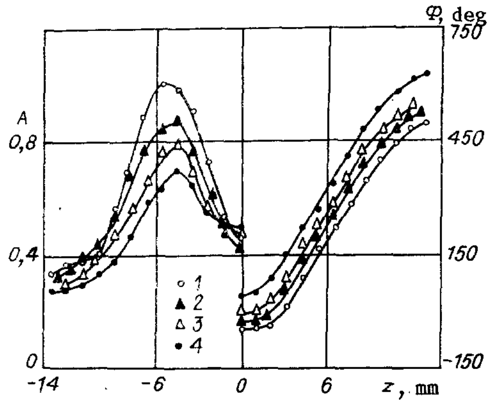

Fig. 1

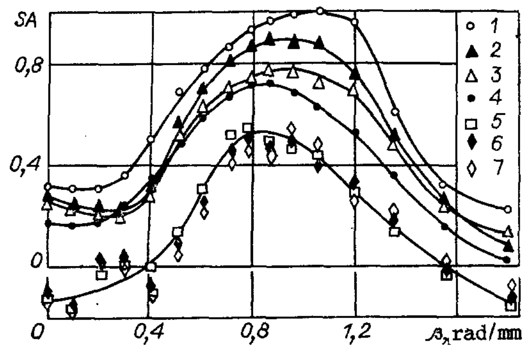

Fig. 2

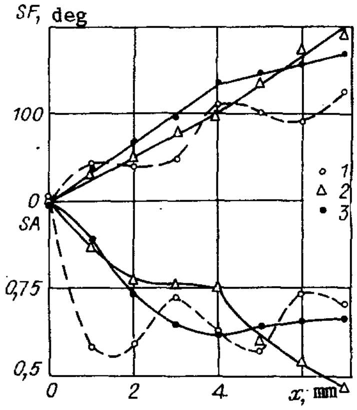

Fig. 3

nate) with an accuracy of ±1 mm. A special mechanism rotated the model relative to the long axis through the angle $\varphi = \pm 100°$, thus changing the transverse coordinate z. The sensor of the anemometer was placed in the holder of the wall-mounted traversing gear and could be moved perpendicular to the flow axis by changing the normal coordinate y to within ±0.01 mm.

The stability characteristics were studied by the method of local disturbance of a supersonic laminar boundary layer [6]. Perturbations of fixed frequency f were introduced into the boundary layer by means of a spark discharge through a 0.5-mm diameter hole in the surface of the model 147 mm from the vertex of the cone. We recorded the distributions of the amplitudes A(z) and phases $\Phi(z)$ of the anemometer signal, which was proportional to the mass-rate fluctuations. The measurements were made for several sections along x. The x coordinate was reckoned from the inflection point on the model surface. The resulting distributions were entered into a MERA-60 minicomputer and were subjected to Fourier analysis. As a result, we obtained the amplitude and phase spectra for the wave numbers β (β is the transverse component of the wave vector).

2. Measurements of characteristics of the average flow made earlier on the same model under the same test conditions showed that there are sections with a negative pressure gradient ($0 < x < 3$ mm) and positive gradient ($4 \text{ mm} < x < 7$ mm) when a laminar boundary layer interacts with a fan of rarefaction waves. The nongradient section of the flow begins at $x \approx 7$ mm. Similarity is restored at the distance $\approx 15\delta_0$ (δ_0 is the thickness of the boundary layer at the inflection point on the surface). For the conditions of the experiment, $\delta_0 = 0.72$ mm. Measurements of the integral characteristics of the boundary layer (displacement thickness and momentum thickness) allowed us to compare the boundary layer on the cylindrical part of the model (where flow was self-similar) with the boundary layer on the flat plate. In the present case, the coordinate x = 15 mm on the cylinder corresponds to Re = 715 on the flat plate, where $Re = (Re_1 x_\ell)^{1/2}$ (x_ℓ is the longitudinal coordinate on the plate, reckoned from the leading edge).

Information on the stability of a laminar boundary layer in a fan of rarefaction waves and behind the fan was obtained in a study of the development of "artificial" perturbations

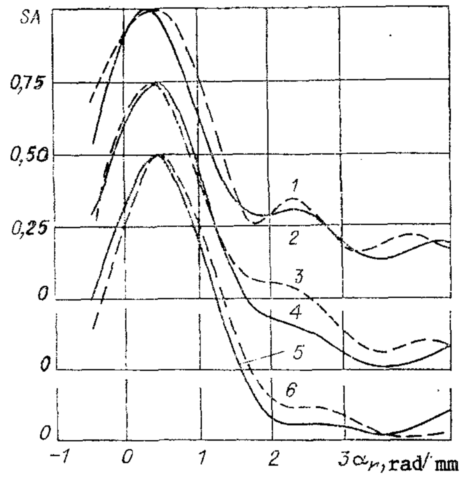

Fig. 4

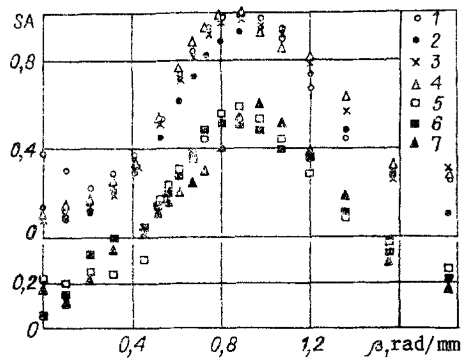

Fig. 5

for $F = 0.38 \cdot 10^{-4}$ (f = 20 kHz). Here, $F = 2\pi f/Re_1 U$ is the dimensionless frequency parameter; f is the dimensional frequency; U is the velocity in the undisturbed flow. The frequency was chosen so that the perturbations in the nongradient boundary layer on the flat plate at Re = 715 would be maximally unstable [6].

Figure 1 presents examples of distributions of the amplitude and phase of the signal introduced into the boundary layer for the sections x = 0, 1, 2, and 4 mm (points 1-4). Similar distributions were found for sections x = 3, 5, 6, and 7 mm. An increase in the distance from the inflection point is accompanied by a reduction in the maximum amplitude in the distributions on the section corresponding to a negative pressure gradient. This indicates that the acceleration of the flow by the perturbation has a stabilizing effect. More detailed information was obtained from the wave spectra after the Fourier analysis.

Figure 2 shows the form of the amplitude spectra of the wave numbers β for x = 0, 1, 2, 4, 5, 6, 7 mm (points 1-7). The perturbations decay for nearly all β (and, accordingly, nearly all wave angles) in the region of x from 0-4 mm (i.e., in the region of the negative pressure gradient).

In the region of the positive pressure gradient (x = 5, 6, 7 mm), the perturbations remain nearly at the same level for all wave angles, although the destabilizing effect of a positive pressure gradient is well known. This result can evidently be explained by the fact that the boundary layer remains stable under the influence of the fan of rarefaction waves in the given case.

We used the amplitude and phase spectra obtained to construct the dependence of the amplitude SA and phase SF on the longitudinal coordinate for different β. The form of these rise curves for β = 0.21, 0.56, and 1.20 (points 1-3) is shown in Fig. 3.

For small β and, accordingly, small wave angles, these curves are distinctly modulatory in character. Thus, it can be suggested that perturbations having a complex structure develop in the boundary layer in the region of interaction with the fan of rarefaction waves. This conclusion is corroborated by the results of the Fourier analysis along x, i.e., the spectra of the wave numbers α_r shown in Fig. 4 (lines 1-6 correspond to β = 0, 0.21, 0.31, 0.44, 0.56, 1.20).

We note the existence of several peaks, which correspond to different types of waves: the largest peak corresponds to the natural wave of the boundary layer, propagating at the phase velocity $c_x \approx 0.5$; the smaller peaks correspond to sound waves with $c_x < 0.5$. It should be noted that the fraction of acoustic perturbations ($\alpha_r \approx 2.5$ and 4) is considerably larger for small wave angles (β < 0.4) than for nongradient flows [6]. Whereas the percentage of sound is about 10% of the Tollman–Schlichting wave for the plate, in the present case it is approximately 20-30%. An increase in the wave angle is accompanied by an initial reduction in the percentage of acoustic perturbations (to β ≈ 0.6-0.7) and its subsequent increase.

It is evident from the rise curves for different β (Fig. 3) that the phase slope is different for sections with positive and negative pressure gradients, which is evidence of the different phase velocities of the perturbations that develop on these sections.

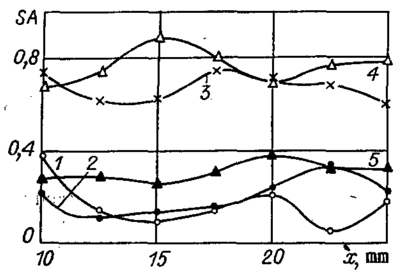

Fig. 6

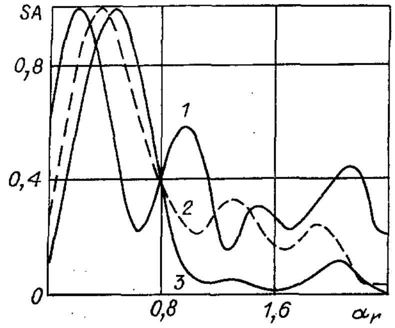

Fig. 7

Due to the effect of the acoustic perturbations, evaluating the phase velocity for small β and low degrees of amplification of the perturbations requires special methods for analyzing the experimental data that would make it possible to distinguish different types of waves.

It was noted in [5] that the stability which develops under the influence of rotation is maintained over large distances after the rotation is completed. This is true even in the case of the nongradient section. To confirm this proposition, we studied the development of perturbations on the cylindrical part of the model after rotation, beginning with x = 10 mm and ending at x = 25 mm. The distributions of the amplitudes and phases of the artificial signal were recorded with an interval of 2.5 mm for the frequency parameter $F = 0.38 \cdot 10^{-4}$. Figure 5 shows the amplitude wave spectra for β in this case (points 1-7 correspond to x = 10.0, 12.5, 15.0, 17.5, 20.0, 22.5, and 25.0 mm).

Whereas the form of the amplitude spectra could be used to readily judge whether perturbations were increasing or decreasing for the cases examined earlier (flat plate [6], cone [7], flow rotation), no such conclusions can be made here. The perturbations remain at roughly the same level, oscillating to one side and the other. The rise curves for β = 0, 0.21, 0.61, 1.20, and 1.56 (lines 1-5) are shown in Fig. 6. It can be concluded from the form of these curves that the perturbations are close to neutral in the investigated range of experimental parameters [the question of oscillations in the relations A(x) in Fig. 6 is examined below]. At the same time, the growth of the perturbations is maximal for a flat plate at Re = 715. Thus, although the boundary layer on the cylindrical part of the investigated model at x = 15 mm has the same velocity profiles and integral thicknesses as the plate at Re = 715, its stability cannot be evaluated on the basis of the locally uniform approximation. It can be seen from the results of the present study that the history of the flow is second in importance only to the rotation. The stability reserve acquired in the interaction of a laminar boundary layer with a fan of rarefaction waves leads to a situation whereby perturbations on the nongradient section of the flow near the rotation remain neutral.

Another interesting feature of the graphs shown in Fig. 6 is that their modulatory character is more pronounced than with flow rotation. This indicates the existence of waves with different phase velocities in the process being investigated. Results of the Fourier analysis along x (wave spectra of α_r) of the distributions shown in Fig. 6 are presented in Fig. 7 for β = 0, 0.21, and 0.61 (lines 1-3). It is evident that the perturbations have a more complicated structure than in the previous case. The percentage of acoustic perturbations is even greater for small wave angles (increasing to about 35%). We should point out the singularity in the spectrum of α_r for β = 0. The highest percentage of energy corresponds to waves with $\alpha_r \approx 0.2$, which corresponds to the phase velocity $c_x \approx 1.1$-1.2 in the interval $1 < c_x < 1 + 1/M$. Mack [8] termed such perturbations regular due to the absence of singular points in the stability equations. Perturbations of this type have not been seen before experimentally. Their appearance might be connected with the passage of the disturbances through the fan of rarefaction waves. With an increase in β, the peak corresponding to the regular perturbations begins to approach the peak corresponding to the Tollman–Schlichting waves. At large β, the first peak is no longer seen.

Thus, the perturbations which develop on the cylindrical part of the model have a complex wave structure. Tollman–Schlichting waves and acoustic perturbations are joined by regular perturbations with a zero wave angle.

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STUDY OF A THREE-DIMENSIONAL TURBULENT BOUNDARY LAYER WITH ALLOWANCE FOR COUPLED HEAT TRANSFER

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UDC 533.526+536.24

This article examines the solution of a problem concerning the heating of a cone with a spherical blunting in a supersonic air flow at angles of attack in the case where the Reynolds numbers are such as to realize different flow regimes. We study the effect of nonisothermality of the surface of the body on the heat flow reaching it in the turbulent boundary layer, and we evaluate the accuracy of conventional approaches based on calculation of heating with a specified coefficient of heat transfer from the gas phase.

1. In accordance with [1, 2], characteristics of coupled heat transfer will be sought from the solution of a system of equations describing the change in the averaged quantities in a three-dimensional boundary layer [3] and the nonsteady unidimensional equation of heat conduction in the skin of a body with corresponding boundary and initial conditions.

The boundary layer on the spherical part of the body was calculated as being axisymmetric in the coordinate system connected with the stagnation point. We then changed over to a semigeodesic coordinate system connected with the symmetry axis of the body. After the introduction of the stream functions f and φ, the system of equations of the three-dimensional boundary layer appears as follows in Dorodnitsyn–Lees coordinates:

$$\frac{\partial}{\partial\zeta}\left(l\frac{\partial\bar{u}}{\partial\zeta}\right)+(\alpha_4 f+\alpha_3\varphi)\frac{\partial\bar{u}}{\partial\zeta}=\alpha_1\left(\bar{u}\frac{\partial\bar{u}}{\partial\xi}-\frac{\partial f}{\partial\xi}\frac{\partial\bar{u}}{\partial\zeta}\right)+\alpha_2\left(\bar{\omega}\frac{\partial\bar{u}}{\partial\eta}-\frac{\partial\varphi}{\partial\eta}\frac{\partial\bar{u}}{\partial\zeta}\right)+ \tag{1.1}$$

$$+\beta_1\left(\bar{u}^2-\frac{\rho_e}{\rho}\right)+\beta_2\left(\bar{\omega}^2-\frac{\rho_e}{\rho}\right)+\beta_3\left(\overline{u\omega}-\frac{\rho_e}{\rho}\right);$$

$$\frac{\partial}{\partial\zeta}\left(l\frac{\partial\bar{\omega}}{\partial\zeta}\right)+(\alpha_4 f+\alpha_3\varphi)\frac{\partial\bar{\omega}}{\partial\zeta}=\alpha_1\left(\bar{u}\frac{\partial\bar{\omega}}{\partial\xi}-\frac{\partial f}{\partial\xi}\frac{\partial\bar{\omega}}{\partial\zeta}\right)+\alpha_2\left(\bar{\omega}\frac{\partial\bar{\omega}}{\partial\eta}-\frac{\partial\varphi}{\partial\eta}\frac{\partial\bar{\omega}}{\partial\zeta}\right)+ \tag{1.2}$$

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